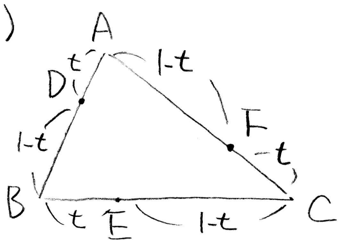
京大理系　1963前期　④

(イ)




A,B,C,D,E,Fの位置ベクトルを $\vec{a},\vec{b},\vec{c},\vec{d},\vec{e},\vec{f}$ とすると,

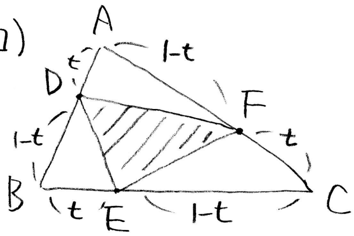
$\vec{d}=\vec{a}+t\overrightarrow{AB}=\vec{a}+t(\vec{b}-\vec{a})=(1-t)\vec{a}+t\vec{b}$

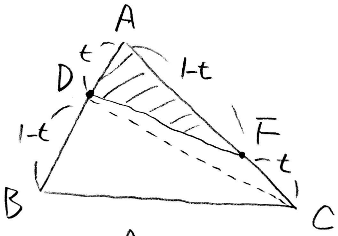
$\vec{e}=\vec{b}+t\overrightarrow{BC}=\vec{b}+t(\vec{c}-\vec{b})=(1-t)\vec{b}+t\vec{c}$

$\vec{f}=\vec{c}+t\overrightarrow{CA}=\vec{c}+t(\vec{a}-\vec{c})=(1-t)\vec{c}+t\vec{a}$

$\dfrac{\vec{d}+\vec{e}+\vec{f}}{3}=\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$ より,題意は示された.

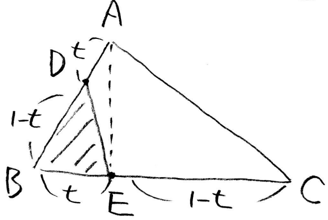
(ロ)



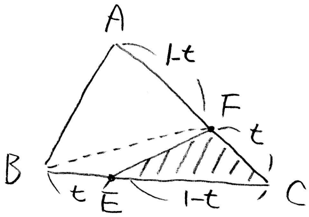

$\dfrac{\triangle ACD}{\triangle ABC}=\dfrac{t}{1}$, $\dfrac{\triangle ADF}{\triangle ACD}=\dfrac{1-t}{1}$ より

$\dfrac{\triangle ADF}{\triangle ABC}=t(1-t)$




$\dfrac{\triangle ABE}{\triangle ABC}=\dfrac{t}{1}$, $\dfrac{\triangle BDE}{\triangle ABE}=\dfrac{1-t}{1}$ より

$\dfrac{\triangle BDE}{\triangle ABC}=t(1-t)$




$\dfrac{\triangle BCF}{\triangle ABC}=\dfrac{t}{1}$. $\dfrac{\triangle CEF}{\triangle BCF}=\dfrac{1-t}{1}$ より

$\dfrac{\triangle CEF}{\triangle ABC}=t(1-t)$

よって $\dfrac{\triangle DEF}{\triangle ABC}=\dfrac{1-3t(1-t)}{1}=3t^2-3t+1=3\left(t^2-t+\dfrac{1}{4}\right)+\dfrac{1}{4}=3\left(t-\dfrac{1}{2}\right)^2+\dfrac{1}{4}$ より.

$\dfrac{1}{4}$倍.