

$$\sin \alpha \cos \alpha + \cos \alpha \sin \alpha + \sin \alpha \cos \beta + \cos \alpha \sin \beta = \sqrt{3} \sin \alpha \quad \text{--- (1)}$$

$$\sin \alpha \neq 0 \text{ のとき, (1) より } \cos \alpha + \sin \alpha \frac{\cos \alpha}{\sin \alpha} + \cos \beta + \sin \beta \frac{\cos \alpha}{\sin \alpha} = \sqrt{3} \quad \text{--- (2)}$$

$$\pm \text{に } \cos \alpha \neq 0 \text{ のとき (2) より } \cos \alpha + \cos \beta + \frac{\sin \alpha + \sin \beta}{\tan \alpha} = \sqrt{3}, \quad \sin \alpha + \sin \beta = (\sqrt{3} - \cos \alpha - \cos \beta) \tan \alpha \quad \text{--- (3)}$$

$$\text{(3) を任意の } \alpha \text{ により成立せよとは } \begin{cases} \sin \alpha + \sin \beta = 0 & \text{--- (4)} \\ \cos \alpha + \cos \beta = \sqrt{3} & \text{--- (5)} \end{cases}$$

$$\text{(4) より } -\frac{\pi}{2} < \alpha < \frac{\pi}{2}, -\frac{\pi}{2} < \beta < \frac{\pi}{2} \text{ より } \beta = -\alpha.$$

$$\text{(5) より } \cos \alpha = \frac{\sqrt{3}}{2}, \quad \alpha = \frac{\pi}{6} \text{ 又は } \alpha = -\frac{\pi}{6}, \quad \beta = -\frac{\pi}{6} \text{ 又は } \beta = \frac{\pi}{6}.$$

$$\cos \alpha = 0 \text{ のとき (2) は } \cos \alpha + \cos \beta = \sqrt{3} \text{ とおき } \alpha = \frac{\pi}{2}, \beta = -\frac{\pi}{6} \text{ 又は } \alpha = -\frac{\pi}{2}, \beta = \frac{\pi}{6} \text{ のとき成立せよ.}$$

$$\sin \alpha = 0 \text{ のとき (1) は } \sin \alpha \cos \alpha + \sin \beta \cos \alpha = 0 \text{ とおき } \alpha = \frac{\pi}{6}, \beta = -\frac{\pi}{6} \text{ 又は } \alpha = -\frac{\pi}{6}, \beta = \frac{\pi}{6} \text{ のとき成立せよ.}$$

$$\text{よって } \alpha = \frac{\pi}{6}, \beta = -\frac{\pi}{6} \text{ 又は } \alpha = -\frac{\pi}{6}, \beta = \frac{\pi}{6}$$