京大理系　1965前期　⑤

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

(ア) $$\lim_{x\to 1}\frac{\frac{1}{x^3}-1}{x-1}=\lim_{x\to 1}\frac{-(x^3-1)}{x^3(x-1)}=\lim_{x\to 1}\frac{-(x-1)(x^2+x+1)}{x^3(x-1)}=-3$$

(イ) $$\lim_{x\to 1}\frac{\sqrt{1+x+x^2}-\sqrt{3}}{x-1}=\lim_{x\to 1}\frac{(\sqrt{1+x+x^2}-\sqrt{3})(\sqrt{1+x+x^2}+\sqrt{3})}{(x-1)(\sqrt{1+x+x^2}+\sqrt{3})}=\lim_{x\to 1}\frac{1+x+x^2-3}{(x-1)(\sqrt{1+x+x^2}+\sqrt{3})}$$

$$=\lim_{x\to 1}\frac{x^2+x-2}{(x-1)(\sqrt{1+x+x^2}+\sqrt{3})}=\lim_{x\to 1}\frac{(x-1)(x+2)}{(x-1)(\sqrt{1+x+x^2}+\sqrt{3})}=\frac{3}{2\sqrt{3}}=\frac{\sqrt{3}}{2}$$

$$\begin{array}{r|l} & x+2 \\ x-1 & x^2+x-2 \\ & x^2-x \\ \hline & 2x-2 \\ & 2x-2 \\ \hline & 0 \end{array}$$