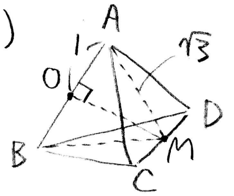
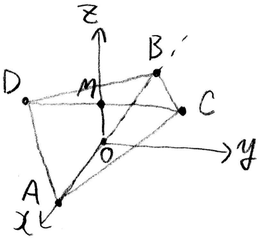
京大理系　1988前期 ④

(1)





$A(1, 0, 0)$, $B(-1, 0, 0)$とする

ABを含みMを含む平面はxz平面

$AM=BM$より$\triangle MAB$は2等辺3角形

左上図より $OM^2+1=3$, $OM=\sqrt{2}$, $M(0, 0, \sqrt{2})$

$BM\perp CD$, $AM\perp CD$より

CDは $\overrightarrow{AM}\times\overrightarrow{BM}=(-1, 0, \sqrt{2})\times(1, 0, \sqrt{2})=(0, 2\sqrt{2}, 0)$に平行

Cのy座標は正であるから $C(0, 1, \sqrt{2})$, $D(0, -1, \sqrt{2})$

$$\begin{array}{cccc} -1 & 0 & \sqrt{2} & -1 \\ 1 & 0 & \sqrt{2} & 1 \\ \hline 0 & 0 & 2\sqrt{2} & \end{array}$$

(2) ACの方程式は $(1, 0, 0)+s(-1, 1, \sqrt{2})=(-s+1, s, \sqrt{2}s)$

αとACの交点は $\sqrt{2}s=\sqrt{2}t$, $s=t$より $(-t+1, t, \sqrt{2}t)$

対称性より αと, ADの交点は$(-t+1, -t, \sqrt{2}t)$, BCの交点は$(t-1, t, \sqrt{2}t)$, BDの交点は$(t-1, -t, \sqrt{2}t)$

よって2辺の長さが $-2t+2$, $2t$ の長方形

(3) $z=u$とACの交点は $\sqrt{2}s=u$, $s=\frac{u}{\sqrt{2}}$より $\left(-\frac{u}{\sqrt{2}}+1, \frac{u}{\sqrt{2}}, u\right)$

対称性より $z=u$と, ADの交点は$\left(-\frac{u}{\sqrt{2}}+1, -\frac{u}{\sqrt{2}}, u\right)$, BCの交点は$\left(\frac{u}{\sqrt{2}}-1, \frac{u}{\sqrt{2}}, u\right)$, BDの交点は$\left(\frac{u}{\sqrt{2}}-1, -\frac{u}{\sqrt{2}}, u\right)$

$z=u$による正四面体ABCDの切り口の面積は $(-\sqrt{2}u+2)\sqrt{2}u=(-2u^2+2\sqrt{2}u)$

求める体積は $\int_0^{\sqrt{2}t}(-2u^2+2\sqrt{2}u)du=\left[-2\frac{u^3}{3}+\cancel{2}\sqrt{2}\frac{u^2}{\cancel{2}}\right]_0^{\sqrt{2}t}=-\frac{4\sqrt{2}}{3}t^3+2\sqrt{2}t^2$