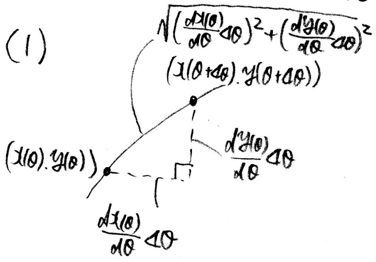
京大理系 1994前期 ⑥

(1)



$$L=\int_0^{2\pi}\sqrt{(-2\sin\theta+\sin2\theta\cdot2)^2+(2\cos\theta-\cos2\theta\cdot2)^2}\,d\theta=2\int_0^{2\pi}\sqrt{\sin^2\theta-2\sin\theta\sin2\theta+\sin^2 2\theta+\cos^2\theta-2\cos\theta\cos2\theta+\cos^2 2\theta}\,d\theta$$

$$=2\sqrt2\int_0^{2\pi}\sqrt{1-\sin\theta\cdot2\sin\theta\cos\theta-\cos\theta(\cos^2\theta-\sin^2\theta)}\,d\theta=2\sqrt2\int_0^{2\pi}\sqrt{1-2(1-\cos^2\theta)\cos\theta-\cos^3\theta+\cos\theta(1-\cos^2\theta)}\,d\theta$$

$$=2\sqrt2\int_0^{2\pi}\sqrt{1-2\cos\theta+\cancel{2\cos^3\theta}-\cancel{\cos^3\theta}+\cos\theta-\cancel{\cos^3\theta}}\,d\theta=2\sqrt2\int_0^{2\pi}\sqrt{2\sin^2\frac{\theta}{2}}\,d\theta=4\int_0^{2\pi}\sin\frac{\theta}{2}\,d\theta$$

※ $\sin^2\frac{\theta}{2}=\frac{1-\cos\theta}{2}$

$$=4\left[-2\cos\frac{\theta}{2}\right]_0^{2\pi}=-8\{(-1)-1\}=16$$

(2) 曲線の$0\leqq\theta\leqq\theta_n$の部分の長さは(1)より $-8\left[\cos\frac{\theta}{2}\right]_0^{\theta_n}=-8\left(\cos\frac{\theta_n}{2}-1\right)$

よって $\cancel{8}\left(1-\cos\frac{\theta_n}{2}\right)=\frac{\cancel{16}^{\,2}}{n}$ $\quad\cancel{2}\sin^2\frac{\theta_n}{4}=\frac{2}{n}$ $\quad\sqrt{n}=\frac{1}{\sin\frac{\theta_n}{4}}$

また、$n\to\infty$のとき、$\theta_n\to0$

ゆえに $\displaystyle\lim_{n\to\infty}\sqrt{n}\,\theta_n=\lim_{\theta_n\to0}\frac{\theta_n}{\sin\frac{\theta_n}{4}}=\lim_{\theta_n\to0}\frac{\frac{\theta_n}{4}}{\sin\frac{\theta_n}{4}}4=4$