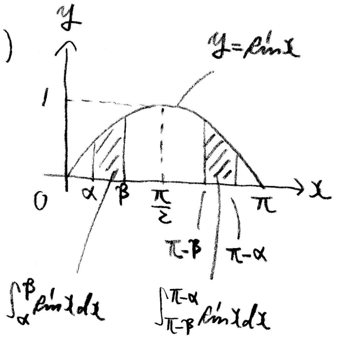
京大理系　1997前期　④

(1)



左図より $\int_\alpha^\beta \sin x\,dx = \int_{\pi-\beta}^{\pi-\alpha} \sin x\,dx$

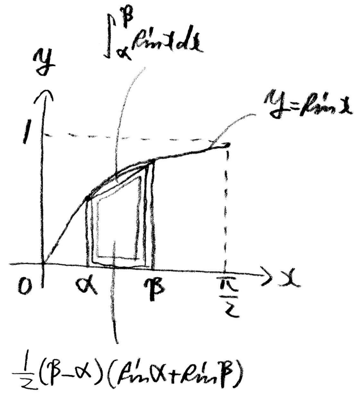
$\sin(\pi-\beta) = -\cos\pi\cdot\sin\beta = \sin\beta$

よって

$\int_\alpha^\beta \sin x\,dx > \frac{1}{2}(\beta-\alpha)(\sin\alpha+\sin\beta)$

を示せばよい.

右図より. これは成り立つ.



(2) (1)より

$\int_0^\pi \sin x\,dx$

$= \int_0^{\frac{\pi}{8}} \sin x\,dx + \int_{\frac{7}{8}\pi}^{\pi} \sin x\,dx + \int_{\frac{\pi}{8}}^{\frac{2}{8}\pi} \sin x\,dx + \int_{\frac{6}{8}\pi}^{\frac{7}{8}\pi} \sin x\,dx + \int_{\frac{2}{8}\pi}^{\frac{3}{8}\pi} \sin x\,dx + \int_{\frac{5}{8}\pi}^{\frac{6}{8}\pi} \sin x\,dx + \int_{\frac{3}{8}\pi}^{\frac{4}{8}\pi} \sin x\,dx + \int_{\frac{4}{8}\pi}^{\frac{5}{8}\pi} \sin x\,dx$

$> \frac{\pi}{8}(\sin 0 + \sin\frac{7}{8}\pi) + \frac{\pi}{8}(\sin\frac{\pi}{8} + \sin\frac{6}{8}\pi) + \frac{\pi}{8}(\sin\frac{2}{8}\pi + \sin\frac{5}{8}\pi) + \frac{\pi}{8}(\sin\frac{3}{8}\pi + \sin\frac{4}{8}\pi)$

$= \frac{\pi}{8}\sum_{k=1}^{7} \sin\frac{k\pi}{8}$ —①

$\int_0^\pi \sin x\,dx = \left[-\cos x\right]_0^\pi = -(-1) - (-1) = 2$ —②

①②より　$2 > \frac{\pi}{8}\sum_{k=1}^{7}\sin\frac{k\pi}{8}$,　$\sum_{k=1}^{7}\sin\frac{k\pi}{8} < \frac{16}{\pi}$