京大理系 1997前期 ⑤

(1) 1が出る確率 p，3が出る確率 $1-p$

1回目	2回目	3回目	確率	得点
1	1	1	p^3	3
1	1	3	$p^2(1-p)$	5
1	3	1	$2p(1-p)$	4
3	1	3	$(1-p)^2$	6
3	3			

左表より $E_A=3p^3+5p^2(1-p)+4\cdot 2p(1-p)+6(1-p)^2$

$=3p^3+5p^2-5p^3+8p-8p^2+6-12p+6p^2$

$=-2p^3+3p^2-4p+6$

1回目	2回目	3回目	確率	得点
1	1	1	p^3	3
1	1	3	$p^2(1-p)$	5
1	3	1	$2p^2(1-p)$	5
1	3	3	$2p(1-p)^2$	0
3	1	3	$(1-p)^2$	6
3	3			

左表より $E_B=3p^3+5p^2(1-p)+5\cdot 2p^2(1-p)+6(1-p)^2$

$=3p^3+5p^2-5p^3+10p^2-10p^3+6-12p+6p^2$

$=-12p^3+21p^2-12p+6$

$E_A-E_B=10p^3-18p^2+8p=2p(5p^2-9p+4)=2p\left(p-\frac{4}{5}\right)(p-1)$ より $E_A-E_B>0$ $E_A>E_B$ のとき $0<p<\frac{4}{5}$

※ $5p^2-9p+4=0$ のとき $p=\frac{9\pm\sqrt{81-80}}{10}=\frac{9\pm1}{10}=\frac{4}{5},1$

(2)

A＼B	3	5	5	0	6
3		B	B	A	B
5	A			A	B
4	A	B	B	A	B
6	A	A	A	A	

上表より $P_A=p^3\cdot 2p(1-p)^2+p^2(1-p)\{p^3+2p(1-p)^2\}+2p(1-p)\{p^3+2p(1-p)^2\}+(1-p)^2\{1-(1-p)^2\}$

$=2p^4-4p^5+2p^6+(-p^3-p^2+2p)(3p^3-4p^2+2p)+(1-2p+p^2)(2p-p^2)$

$=2p^4-4p^5+2p^6-3p^6+4p^5-2p^4-3p^5+4p^4-2p^3+6p^4-8p^3+4p^2+2p-p^2-4p^2+2p^3+2p^3-p^4$

$=-p^6-3p^5+9p^4-6p^3-p^2+2p$

$P_B=p^3\{p^2(1-p)+2p^2(1-p)+(1-p)^2\}+p^2(1-p)(1-p)^2+2p(1-p)\{p^2(1-p)+2p^2(1-p)+(1-p)^2\}$

$=(p^3-2p^2+2p)(p^2-p^3+2p^3-2p^2+1-2p+p^2)+p^2(1-3p+3p^2-p^3)$

$=(p^3-2p^2+2p)(-3p^3+4p^2-2p+1)+p^2-3p^3+3p^4-p^5$

$=-3p^6+4p^5-2p^4+p^3+6p^5-8p^4+4p^3-2p^2-6p^4+8p^3-4p^2+2p+p^2-3p^3+3p^4-p^5$

$=-3p^6+9p^5-13p^4+10p^3-5p^2+2p$

$P_A-P_B=2p^6-12p^5+22p^4-16p^3+4p^2=2p^2(p^4-6p^3+11p^2-8p+2)$

$=2p^2(p-1)^2\{p-(2-\sqrt{2})\}\{p-(2+\sqrt{2})\}$

より $P_A>P_B$ のとき $0<p<2-\sqrt{2}$

よって、いえない

p^3-5p^2+6p-2

$p-1\overline{)\,p^4-6p^3+11p^2-8p+2}$

p^4-p^3

$-5p^3+11p^2$

$-5p^3+5p^2$

$6p^2-8p$

$6p^2-6p$

$-2p+2$

$-2p+2$

0

p^2-4p+2

$p-1\overline{)\,p^3-5p^2+6p-2}$

p^3-p^2

$-4p^2+6p$

$-4p^2+4p$

$2p-2$

$2p-2$

0

$p^2-4p+2=0$ のとき

$p=2\pm\sqrt{4-2}=2\pm\sqrt{2}$