京大理系 1999前期 ⑥

$$x' = \frac{(3-2t)(t+1)-(3t-t^2)}{(t+1)^2} = \frac{\cancel{3t}+3-2t^2-2t\cancel{-3t}+t^2}{(t+1)^2} = \frac{-t^2-2t+3}{(t+1)^2}$$

$x'=0$ のとき $t^2+2t-3=0$. $t=-1\pm\sqrt{1+3}=-1\pm 2=-3, 1$

t	0	$\cdots$	1	$\cdots$	3
x'		$+$	0	$-$	
x	0	↗	1	↘	0

xの増減表は左表

よって $0 \leqq x \leqq 1$

$$y' = \frac{(6t-3t^2)(t+1)-(3t^2-t^3)}{(t+1)^2} = \frac{\cancel{6t^2}+6t-3t^3\cancel{-3t^2}\cancel{-3t^2}+t^3}{(t+1)^2} = \frac{-2t(t^2-3)}{(t+1)^2}$$

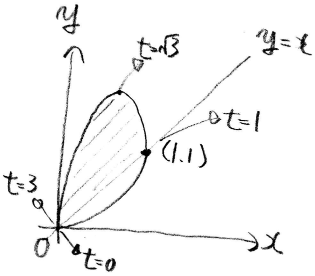
$y'=0$ のとき. $t=\pm\sqrt{3}, 0$

t	0	$\cdots$	$\sqrt{3}$	$\cdots$	3
y'		$+$	0	$-$	
y	0	↗	$6\sqrt{3}-9$	↘	0

yの増減表は左表

よって $0 \leqq y \leqq 6\sqrt{3}-9$

※ $y(\sqrt{3}) = \frac{9-3\sqrt{3}}{\sqrt{3}+1} = \frac{3(3-\sqrt{3})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3}{2}(3\sqrt{3}-3-3+\sqrt{3}) = 6\sqrt{3}-9$



$$y-x = \frac{3t^2-t^3-3t+t^2}{t+1} = \frac{-t(t^2-4t+3)}{t+1} = \frac{-t(t-1)(t-3)}{t+1}$$ より

$y \geqq x$ のとき. $1 \leqq t \leqq 3$

(x, y)が描くグラフは右上図のようになる

求める面積をSとすると、Sは右上図の斜線部の面積である

$$S = \int_0^1 y\,dx - \frac{1}{2} = \int_3^1 \frac{-t^2(t-3)}{t+1}\cdot\frac{-(t+3)(t-1)}{(t+1)^2}\,dt - \frac{1}{2} = -\int_1^3 \frac{(t+3)t^2(t-1)(t-3)}{(t+1)^3}\,dt - \frac{1}{2}$$

$x = \frac{3t-t^2}{t+1}$ とおく. $\begin{array}{c|c} x & 0\to 1 \\ t & 3\to 1 \end{array}$, $\frac{dx}{dt} = \frac{-(t+3)(t-1)}{(t+1)^2}$

$t+1=T$ とおく $\begin{array}{c|c} t & 1\to 3 \\ T & 2\to 4 \end{array}$, $\frac{dT}{dt}=1$

$$= -\int_2^4 \frac{(T+2)(T-1)^2(T-2)(T-4)}{T^3}\,dT - \frac{1}{2} = -\int_2^4 \frac{(T^2-4)(T-4)(T^2-2T+1)}{T^3}\,dT - \frac{1}{2} = -\int_2^4 \frac{(T^3-4T^2-4T+16)(T^2-2T+1)}{T^3}\,dT - \frac{1}{2}$$

$$= -\int_2^4 \frac{T^5-2T^4+T^3-4T^4+8T^3-4T^2-4T^3+8T^2-4T+16T^2-32T+16}{T^3}\,dT - \frac{1}{2} = -\int_2^4 \frac{T^5-6T^4+5T^3+20T^2-36T+16}{T^3}\,dT - \frac{1}{2}$$

$$= \int_2^4 \left(-T^2+6T-5-\frac{20}{T}+\frac{36}{T^2}-\frac{16}{T^3}\right)dT - \frac{1}{2} = \left[-\frac{T^3}{3}+\cancel{6}^{3}\frac{T^2}{\cancel{2}}-5T-20\log T+36\left(-\frac{1}{T}\right)-16\left(-\frac{1}{2T^2}\right)\right]_2^4 - \frac{1}{2}$$

$$= -\frac{64}{3}+48-20-20\log 4-9+\cancel{\frac{1}{2}}+\frac{8}{3}-12+10+20\log 2+18-2-\cancel{\frac{1}{2}} = -\frac{56}{3}+33-20\log 2 = \frac{99-56}{3}-20\log 2$$

$$= \frac{43}{3}-20\log 2$$