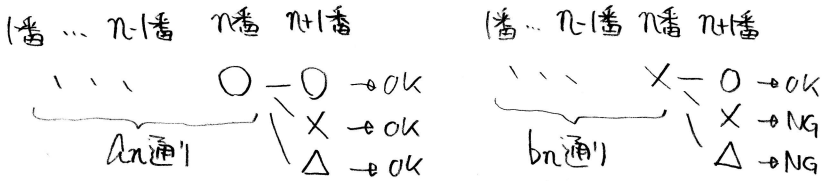
京大理系　2005前期　⑥

赤,青,黄を○,×,△で表す.

題意を満たす塗り方のうち,n番が○の場合の数をa_n,×または△の場合の数をb_nとする.



n=2のとき

○○
○×
○△
×○
△○

n=3のとき

○○○　×○○
○○×　×○×
○○△　×○△
○×○　△○○
○△○　△○×
　　△○△

上図より. $n \geqq 2$のとき $\begin{cases} a_{n+1} = a_n + b_n \\ b_{n+1} = 2a_n \end{cases}$. $a_2 = 3$, $a_3 = 5$

$a_{n+2} = a_{n+1} + 2a_n$, $x^2 = x + 2$ $x^2 - x - 2 = 0$ $x = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = -1, 2$

$a_{n+2} + a_{n+1} = 2(a_{n+1} + a_n)$ より $a_{n+1} + a_n = 2(a_n + a_{n-1}) = 2^2(a_{n-1} + a_{n-2}) = \cdots = 2^{n-2}(a_3 + a_2) = 2^{n+1}$

$a_{n+2} - 2a_{n+1} = -(a_{n+1} - 2a_n)$ より $a_{n+1} - 2a_n = -(a_n - 2a_{n-1}) = (-1)^2(a_{n-1} - 2a_{n-2}) = \cdots = (-1)^{n-2}(a_3 - 2a_2) = (-1)^{n-1}$

$3a_n = 2^{n+1} - (-1)^{n-1}$, $a_n = \frac{1}{3}2^{n+1} - \frac{1}{3}(-1)^{n-1}$

$b_n = a_{n+1} - a_n = \frac{1}{3}2^{n+2} - \frac{1}{3}(-1)^n - \frac{1}{3}2^{n+1} + \frac{1}{3}(-1)^{n-1} = \frac{1}{3}2^{n+1} + \frac{2}{3}(-1)^{n-1}$

以上より. 題意を満たす塗り方は

$\frac{1}{3}2^{n+1} - \frac{1}{3}(-1)^{n-1} + \frac{1}{3}2^{n+1} + \frac{2}{3}(-1)^{n-1} = \frac{2}{3}2^{n+1} + \frac{1}{3}(-1)^{n-1} = \frac{1}{3}2^{n+2} - \frac{1}{3}(-1)^n$ 通り