

京大文系 2009前期A (2)

$$\int_0^x f(y) dy + x^2 \int_0^1 f(y) dy + 2x \int_0^1 y f(y) dy + \int_0^1 y^2 f(y) dy = x^2 + C \quad \text{--- (1)}$$

$$f(x) + 2x \int_0^1 f(y) dy + 2 \int_0^1 y f(y) dy = 2x \quad \text{--- (2)}$$

$$f'(x) + 2 \int_0^1 f(y) dy = 2 \quad \text{--- (3)}$$

(3) f) $f(x)$ は定数と仮定して $f(x) = \alpha x + \beta$ とおこう。

$$(3) f) \alpha + 2 \int_0^1 (\alpha y + \beta) dy = 2, \quad \alpha + 2 \left[\alpha \frac{y^2}{2} + \beta y \right]_0^1 = 2, \quad \alpha + 2 \left(\frac{\alpha}{2} + \beta \right) = 2, \quad \alpha + \beta = 1 \quad \text{--- (3)'}$$

$$(2) \text{で } x=0 \text{ とおくと } \beta + 2 \int_0^1 y (\alpha y + \beta) dy = 0, \quad \beta + 2 \left[\alpha \frac{y^3}{3} + \beta \frac{y^2}{2} \right]_0^1 = 0, \quad \beta + 2 \left(\frac{\alpha}{3} + \frac{\beta}{2} \right) = 0, \quad \frac{2}{3} \alpha + 2\beta = 0, \quad \alpha + 3\beta = 0 \quad \text{--- (2)'}$$

$$(2) (3)' f) \quad -3\beta + \beta = 1, \quad \beta = -\frac{1}{2}, \quad \alpha = \frac{3}{2}, \quad f(x) = \frac{3}{2}x - \frac{1}{2}.$$

$$(1) \text{で } x=0 \text{ とおくと } \int_0^1 y^2 \left(\frac{3}{2}y - \frac{1}{2} \right) dy = C.$$

$$C = \left[\frac{3}{2} \frac{y^4}{4} - \frac{1}{2} \frac{y^3}{3} \right]_0^1 = \frac{3}{8} - \frac{1}{6} = \frac{9-4}{24} = \frac{5}{24}$$