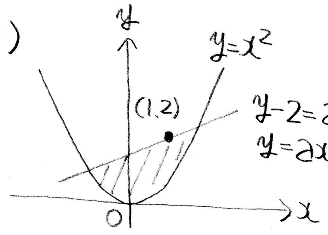
京大文系　2010前期　①

(1)



xについての2次方程式

$x^2=ax-a+2$, $x^2-ax+a-2=0$

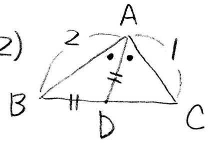
の解をα, β $(\alpha<\beta)$とする.

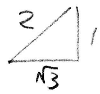
$S(a)=\int_\alpha^\beta(ax-a+2-x^2)dx=-\int_\alpha^\beta(x-\alpha)(x-\beta)dx=\frac{1}{6}(\beta-\alpha)^3$

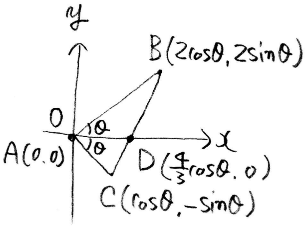
$\alpha+\beta=a$, $\alpha\beta=a-2$ より. $(\beta-\alpha)^2=(\beta+\alpha)^2-4\alpha\beta=a^2-4a+8=(a-2)^2+4$. $S(a)=\frac{1}{6}\{(a-2)^2+4\}^{\frac{3}{2}}$

よって. $a=2$

(2)







xy平面で考えると. A, B, Cの座標は左図のようにとれる.

直線BCの方程式は　$y+\sin\theta=\frac{3\sin\theta}{\cos\theta}(x-\cos\theta)$

$\cos\theta=3x-3\cos\theta$. $x=\frac{4}{3}\cos\theta$ より. Dの座標は $(\frac{4}{3}\cos\theta, 0)$

AD=BDより. $\frac{16}{9}\cos^2\theta=\frac{4}{9}\cos^2\theta+4\sin^2\theta$. $\frac{\cancel{4}}{3}\cos^2\theta=\cancel{4}(1-\cos^2\theta)$. $\frac{4}{3}\cos^2\theta=1$. $\cos^2\theta=\frac{3}{4}$. $\cos\theta=\frac{\sqrt{3}}{2}$

$\theta=\frac{\pi}{6}$. $\sin\theta=\frac{1}{2}$

よって. $B(2\frac{\sqrt{3}}{2}, 2\frac{1}{2})=(\sqrt{3}, 1)$　$C(\frac{\sqrt{3}}{2}, -\frac{1}{2})$. $D(\frac{4}{3}\frac{\sqrt{3}}{2}, 0)=(\frac{2}{3}\sqrt{3}, 0)$

△ABCの面積は. $\frac{1}{\cancel{2}}\frac{2}{3}\sqrt{3}+\frac{1}{\cancel{2}}\frac{2}{3}\sqrt{3}\frac{1}{2}=\frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{6}=\frac{2+1}{6}\sqrt{3}=\frac{\sqrt{3}}{2}$