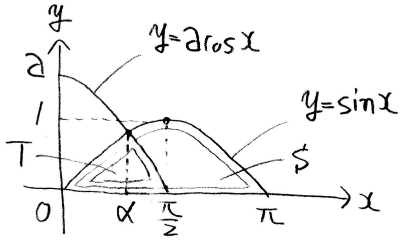
京大理系　2010前期　甲⑤



$y=\sin x$ と $y=a\cos x$ はただ1つの交点を持つから

その x 座標を α とする

$\sin\alpha = a\cos\alpha$

$S=\int_0^{\pi}\sin x\,dx=\left[-\cos x\right]_0^{\pi}=1-(-1)=2$

$T=\int_0^{\alpha}\sin x\,dx+\int_{\alpha}^{\frac{\pi}{2}}a\cos x\,dx=\left[-\cos x\right]_0^{\alpha}+a\left[\sin x\right]_{\alpha}^{\frac{\pi}{2}}=-\cos\alpha-(-1)+a(1-\sin\alpha)$

$=-\cos\alpha+1+\frac{\sin\alpha}{\cos\alpha}-\frac{\sin^2\alpha}{\cos\alpha}=\cancel{-\cos\alpha}+1+\frac{\sin\alpha}{\cos\alpha}-\frac{1}{\cos\alpha}+\cancel{\cos\alpha}$

$\frac{1+\frac{\sin\alpha}{\cos\alpha}-\frac{1}{\cos\alpha}}{2}=\frac{1}{3}$ より　$3+3\frac{\sin\alpha}{\cos\alpha}-3\frac{1}{\cos\alpha}=2$，$\cos\alpha+3\sin\alpha-3=0$　$\cos\alpha=3(1-\sin\alpha)$

両辺は正であるから　$1-\sin^2\alpha=9-18\sin\alpha+9\sin^2\alpha$，$10\sin^2\alpha-18\sin\alpha+8=0$

$(\sin\alpha-1)(10\sin\alpha-8)=0$　$\sin\alpha\neq1$ より　$\sin\alpha=\frac{4}{5}$，$\cos\alpha=\frac{3}{5}$　$a=\frac{\frac{4}{5}}{\frac{3}{5}}=\frac{4}{3}$