

京大理系 2011前期 ①

(1) 京大文系 2011前期 ① (2) を参照 (同じ問題)

$$(2) x = \frac{\sin \theta}{\sqrt{2}} \text{ と } x < \frac{x}{\theta} \begin{cases} 0 \rightarrow \frac{1}{2} \\ 0 \rightarrow \frac{\pi}{4} \end{cases} \quad \frac{dx}{d\theta} = \frac{\cos \theta}{\sqrt{2}}$$

$$\begin{aligned} S_A &= \int_0^{\frac{\pi}{4}} \left( \frac{\sqrt{2}}{2} \sin \theta + 1 \right) \sqrt{1 - 2 \frac{\sin^2 \theta}{2}} \frac{\sqrt{2}}{2} \cos \theta d\theta = \int_0^{\frac{\pi}{4}} \left( \frac{1}{2} \sin \theta \cos^2 \theta + \frac{\sqrt{2}}{2} \cos^2 \theta \right) d\theta \\ &= \int_0^{\frac{\pi}{4}} \left\{ \frac{1}{2} \left( -\frac{1}{3} \cos^3 \theta \right)' + \frac{\sqrt{2}}{2} \frac{1 + \cos 2\theta}{2} \right\} d\theta = \left[ -\frac{1}{6} \cos^3 \theta + \frac{\sqrt{2}}{4} \theta + \frac{\sqrt{2}}{4} \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{4}} \end{aligned}$$

$$* (\cos^3 \theta)' = -3 \cos^2 \theta \sin \theta$$

$$= -\frac{1}{6} \frac{1}{2\sqrt{2}} + \frac{\sqrt{2}}{4} \frac{\pi}{4} + \frac{\sqrt{2}}{8} + \frac{1}{6} = \frac{\sqrt{2}}{16} \pi + \frac{-\sqrt{2} + 3\sqrt{2}}{24} + \frac{1}{6} = \frac{\sqrt{2}}{12} \pi + \frac{\sqrt{2}}{12} + \frac{1}{6}$$