

$Y_n \geq 1$

$\frac{2}{1+\sqrt{3}} = \frac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \sqrt{3}-1 \quad \frac{1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{2}$

(i) $1 \leq Y_n < \frac{1+\sqrt{3}}{2}$ のとき $X_{n+1} + \sqrt{3} - 1 < Y_{n+1} \leq X_{n+1} + 1$ $X_{n+1} = 1$ のとき $\sqrt{3} < Y_{n+1} \leq 2$ $\frac{1+\sqrt{3}}{2} \leq Y_{n+1} \leq 1+\sqrt{3}$
 $X_{n+1} \geq 2$ のとき $Y_{n+1} > 1+\sqrt{3}$

(ii) $\frac{1+\sqrt{3}}{2} \leq Y_n \leq 1+\sqrt{3}$ のとき $X_{n+1} + \frac{\sqrt{3}-1}{2} \leq Y_{n+1} \leq X_{n+1} + \sqrt{3} - 1$ $X_{n+1} = 1$ のとき $\frac{1+\sqrt{3}}{2} \leq Y_{n+1} \leq \sqrt{3}$ $\frac{1+\sqrt{3}}{2} \leq Y_{n+1} \leq 1+\sqrt{3}$
 $X_{n+1} = 2$ のとき $\frac{3+\sqrt{3}}{2} \leq Y_{n+1} \leq 1+\sqrt{3}$ $\frac{1+\sqrt{3}}{2} \leq Y_{n+1} \leq 1+\sqrt{3}$
 $X_{n+1} \geq 3$ のとき $Y_{n+1} \geq \frac{5+\sqrt{3}}{2}$ $Y_{n+1} > 1+\sqrt{3}$

(iii) $Y_n > 1+\sqrt{3}$ のとき $X_{n+1} < Y_{n+1} < X_{n+1} + \frac{\sqrt{3}-1}{2}$ $X_{n+1} = 1$ のとき $1 < Y_{n+1} < \frac{1+\sqrt{3}}{2}$ $1 \leq Y_{n+1} < \frac{1+\sqrt{3}}{2}$
 $X_{n+1} = 2$ のとき $2 < Y_{n+1} < \frac{3+\sqrt{3}}{2}$ $\frac{1+\sqrt{3}}{2} \leq Y_{n+1} \leq 1+\sqrt{3}$
 $X_{n+1} \geq 3$ のとき $Y_{n+1} > 3$ $Y_{n+1} > 1+\sqrt{3}$

$1 \leq Y_n < \frac{1+\sqrt{3}}{2}$ とする確率を a_n , $Y_n > 1+\sqrt{3}$ とする確率を b_n とする $a_n + p_n + b_n = 1$ — (1)

(i)(ii)(iii) より $a_{n+1} = \frac{1}{6} b_n$
 $p_{n+1} = \frac{1}{6} a_n + \frac{1}{3} p_n + \frac{1}{6} b_n$ — (2)
 $b_{n+1} = \frac{5}{6} a_n + \frac{2}{3} p_n + \frac{2}{3} b_n$

①②より $p_{n+1} = \frac{1}{6}(1-p_n) + \frac{1}{3} p_n = \frac{1}{6} p_n + \frac{1}{6}$

$p = \frac{1}{6} p + \frac{1}{6}$ $\frac{5}{6} p = \frac{1}{6}$ $p = \frac{1}{5}$

$n \geq 2$ のとき $p_n - \frac{1}{5} = \frac{1}{6}(p_{n-1} - \frac{1}{5}) = \dots = (\frac{1}{6})^{n-1} (p_1 - \frac{1}{5})$

$p_1 = \frac{1}{6}$ より $p_n = (\frac{1}{6})^{n-1} (\frac{1}{6} - \frac{1}{5}) + \frac{1}{5} = (\frac{1}{6})^{n-1} \frac{5-6}{30} + \frac{1}{5} = -\frac{1}{5} (\frac{1}{6})^{n-1} + \frac{1}{5} = \frac{1}{5} \{ 1 - (\frac{1}{6})^{n-1} \}$ (ただし $n=1$ のときも成立)