東工大　1992前期　⑤

(1)
$$I_n - I_{n-1} = (-1)^n\int_0^{\frac{\pi}{4}} \frac{x\cos(2n-1)x}{\cos x}\,dx - (-1)^{n-1}\int_0^{\frac{\pi}{4}} \frac{x\cos(2n-3)x}{\cos x}\,dx$$
$$= (-1)^n\int_0^{\frac{\pi}{4}} \frac{x\{\cos(2n-1)x+\cos(2n-3)x\}}{\cos x}\,dx$$
$$= (-1)^n\int_0^{\frac{\pi}{4}} \frac{x}{\cancel{\cos x}}\,2\cos\frac{4n-4}{2}x\,\cancel{\cos\frac{2}{2}x}\,dx$$
$$= 2(-1)^n\int_0^{\frac{\pi}{4}} x\cos(2n-2)x\,dx$$
$$= 2(-1)^n\int_0^{\frac{\pi}{4}} x\left\{\frac{\sin(2n-2)x}{2n-2}\right\}'dx$$
$$= 2(-1)^n\left[x\frac{\sin(2n-2)x}{2n-2}\right]_0^{\frac{\pi}{4}} - 2(-1)^n\int_0^{\frac{\pi}{4}} \frac{\sin(2n-2)x}{2n-2}\,dx$$
$$= (-1)^n\frac{1}{n-1}\frac{\pi}{4}\sin\frac{n-1}{2}\pi - (-1)^n\frac{1}{n-1}\left[-\frac{\cos(2n-2)x}{2n-2}\right]_0^{\frac{\pi}{4}}$$
$$= (-1)^n\frac{1}{n-1}\frac{\pi}{4}\sin\frac{n-1}{2}\pi + (-1)^n\frac{1}{(n-1)^2}\frac{1}{2}\left(\cos\frac{n-1}{2}\pi - 1\right)$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$+)\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\cos(\alpha+\beta)+\cos(\alpha-\beta) = 2\cos\alpha\cos\beta$$

$$\alpha+\beta = A \qquad \alpha+\beta = A$$
$$+)\ \alpha-\beta = B \qquad -)\ \alpha-\beta = B$$
$$2\alpha = A+B \qquad 2\beta = A-B$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

(2)
$$I_3 - I_2 = \cancel{(-1)^3\frac{1}{2}\frac{\pi}{4}\sin\pi} + (-1)^3\frac{1}{4}\frac{1}{2}(\cos\pi - 1) = -\frac{1}{8}(-2) = \frac{1}{4}$$
$$I_2 - I_1 = (-1)^2\frac{\pi}{4}\sin\frac{\pi}{2} + (-1)^2\frac{1}{2}\left(\cancel{\cos\frac{\pi}{2}} - 1\right) = \frac{\pi}{4} - \frac{1}{2}$$
$$I_1 = -\int_0^{\frac{\pi}{4}} \frac{x\cancel{\cos x}}{\cancel{\cos x}}\,dx = -\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{4}} = -\frac{\pi^2}{32}$$

よって $I_3 = \frac{1}{4} + I_2 = \frac{1}{4} + \frac{\pi}{4} - \frac{1}{2} + I_1 = -\frac{\pi^2}{32} + \frac{\pi}{4} - \frac{1}{4}$