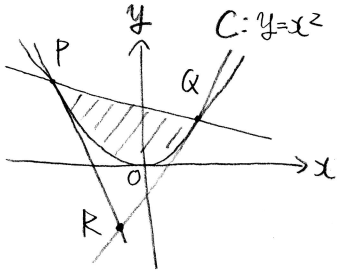
東工大　1994前期 ①



P, Qの座標を (α, α^2), (β, β^2) $(\alpha<\beta)$ とする.

直線PQの方程式は $y-\alpha^2=\frac{\beta^2-\alpha^2}{\beta-\alpha}(x-\alpha)$, $y=(\alpha+\beta)x-\cancel{\alpha^2}-\alpha\beta+\cancel{\alpha^2}$

$$\int_\alpha^\beta\{(\alpha+\beta)x-\alpha\beta-x^2\}dx=\left[-\frac{x^3}{3}+(\alpha+\beta)\frac{x^2}{2}-\alpha\beta x\right]_\alpha^\beta=-\frac{\beta^3}{3}+(\alpha+\beta)\frac{\beta^2}{2}-\alpha\beta^2+\frac{\alpha^3}{3}-(\alpha+\beta)\frac{\alpha^2}{2}+\alpha^2\beta$$

$$=\frac{-2\beta^3+3\beta^2\alpha+3\beta^3-6\beta^2\alpha+2\alpha^3-3\alpha^3-3\beta\alpha^2+6\beta\alpha^2}{6}=\frac{\beta^3-3\beta^2\alpha+3\beta\alpha^2-\alpha^3}{6}=\frac{(\beta-\alpha)^3}{6}=\frac{1}{6}$$ より.

$(\beta-\alpha)^3=1$, $\beta-\alpha=1$ —①

CのPにおける接線の方程式は $y-\alpha^2=2\alpha(x-\alpha)$, $y=2\alpha x-\alpha^2$

同様に, CのQにおける接線の方程式は $y=2\beta x-\beta^2$

$2\alpha x-\alpha^2=2\beta x-\beta^2$, $\cancel{(\beta-\alpha)}(\beta+\alpha)=2\cancel{(\beta-\alpha)}x$, $x=\frac{\alpha+\beta}{2}$, $y=\cancel{2}\alpha\frac{\alpha+\beta}{\cancel{2}}-\alpha^2$, $y=\alpha\beta$ より.

Rの座標は $\left(\frac{\alpha+\beta}{2}, \alpha\beta\right)$

①より, これは $\left(\frac{\alpha+\alpha+1}{2}, \alpha(\alpha+1)\right)=\left(\alpha+\frac{1}{2}, (\alpha+\frac{1}{2})^2-\frac{1}{4}\right)$ となる.

よって, 求める方程式は $y=x^2-\frac{1}{4}$