

(1) $(x-\alpha)(x-\beta) = x^2 + (-\alpha-\beta)x + \alpha\beta \neq 1$. $S = -\alpha-\beta$, $T = \alpha\beta$

S, T の期待値を \bar{S}, \bar{T} とすると. $\bar{S} = \frac{1}{36} \sum_{\alpha=1}^6 \sum_{\beta=1}^6 (-\alpha-\beta)$, $\bar{T} = \frac{1}{36} \sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha\beta$

∴ $\sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha = \sum_{\alpha=1}^6 \sum_{\beta=1}^6 \beta = \sum_{\alpha=1}^6 \alpha \cdot \sum_{\beta=1}^6 1 = \frac{1}{2} \cdot 6 \cdot 7 \cdot 6 = 36 \frac{7}{2}$

$\sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha\beta = \sum_{\alpha=1}^6 \alpha \cdot \sum_{\beta=1}^6 \beta = \frac{1}{2} \cdot 6 \cdot 7 \cdot \frac{1}{2} \cdot 6 \cdot 7 = 36 \frac{49}{4} \neq 1$.

$\bar{S} = \frac{1}{36} (-36 \frac{7}{2} - 36 \frac{7}{2}) = -7$, $\bar{T} = \frac{1}{36} 36 \frac{49}{4} = \frac{49}{4}$

(2) $f(x) = \{x^2 + (-\alpha-\beta)x + \alpha\beta\}^2 = x^4 + (\alpha^2 + 2\alpha\beta + \beta^2)x^2 + \alpha^2\beta^2 + (-2\alpha - 2\beta)x^3 + 2\alpha\beta x^2 + (-2\alpha^2\beta - 2\alpha\beta^2)x$
 $= x^4 + (-2\alpha - 2\beta)x^3 + (\alpha^2 + 4\alpha\beta + \beta^2)x^2 + (-2\alpha^2\beta - 2\alpha\beta^2)x + \alpha^2\beta^2 \neq 1$.

$a = -2\alpha - 2\beta$, $b = \alpha^2 + 4\alpha\beta + \beta^2$, $c = -2\alpha^2\beta - 2\alpha\beta^2$, $d = \alpha^2\beta^2$

a, b, c, d の期待値を A, B, C, D とすると.

$A = \frac{1}{36} \sum_{\alpha=1}^6 \sum_{\beta=1}^6 (-2\alpha - 2\beta)$, $B = \frac{1}{36} \sum_{\alpha=1}^6 \sum_{\beta=1}^6 (\alpha^2 + 4\alpha\beta + \beta^2)$, $C = \frac{1}{36} \sum_{\alpha=1}^6 \sum_{\beta=1}^6 (-2\alpha^2\beta - 2\alpha\beta^2)$, $D = \frac{1}{36} \sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha^2\beta^2$

∴ (1) と, $\sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha^2 = \sum_{\alpha=1}^6 \sum_{\beta=1}^6 \beta^2 = \sum_{\alpha=1}^6 \alpha^2 \cdot \sum_{\beta=1}^6 1 = \frac{1}{6} \cdot 6 \cdot 7 \cdot 13 \cdot 6 = 36 \frac{91}{6}$

$\sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha^2\beta = \sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha\beta^2 = \sum_{\alpha=1}^6 \alpha^2 \cdot \sum_{\beta=1}^6 \beta = \frac{1}{6} \cdot 6 \cdot 7 \cdot 13 \cdot \frac{1}{2} \cdot 6 \cdot 7 = 36 \frac{637}{12}$

$\sum_{\alpha=1}^6 \sum_{\beta=1}^6 \alpha\beta^2 = \sum_{\alpha=1}^6 \alpha^2 \cdot \sum_{\beta=1}^6 \beta^2 = \frac{1}{6} \cdot 6 \cdot 7 \cdot 13 \cdot \frac{1}{6} \cdot 6 \cdot 7 \cdot 13 = 36 \frac{8281}{36} \neq 1$.

$A = \frac{1}{36} (-2 \cdot 36 \frac{7}{2} - 2 \cdot 36 \frac{7}{2}) = -14$

$B = \frac{1}{36} (36 \frac{91}{6} + 4 \cdot 36 \frac{49}{4} + 36 \frac{91}{6}) = \frac{91}{3} + 49 = \frac{238}{3}$

$C = \frac{1}{36} (-2 \cdot 36 \frac{637}{12} - 2 \cdot 36 \frac{637}{12}) = -\frac{637}{3}$

$D = \frac{1}{36} 36 \frac{8281}{36} = \frac{8281}{36}$

49
x13
147
49
637

91
x4
364
317
8281

147
+ 91
238