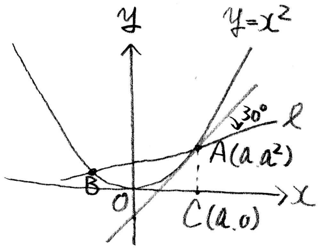
東工大 2007前期 ②

(1)



Aにおける接線の方向ベクトルは $(1,2a)$

lの方向ベクトルは $\begin{pmatrix}\cos(-30°) & -\sin(-30°)\\ \sin(-30°) & \cos(-30°)\end{pmatrix}\begin{pmatrix}1\\2a\end{pmatrix}=\begin{pmatrix}\frac{\sqrt3}{2} & \frac12\\ -\frac12 & \frac{\sqrt3}{2}\end{pmatrix}\begin{pmatrix}1\\2a\end{pmatrix}=\begin{pmatrix}a+\frac{\sqrt3}{2}\\ \sqrt3a-\frac12\end{pmatrix}$

lの傾きは $\dfrac{\sqrt3a-\frac12}{a+\frac{\sqrt3}{2}}=\dfrac{2\sqrt3a-1}{2a+\sqrt3}$

lの式は、$y-a^2=\dfrac{2\sqrt3a-1}{2a+\sqrt3}(x-a)$, $y=\dfrac{2\sqrt3a-1}{2a+\sqrt3}x+\dfrac{-2\sqrt3a^2+a+2a^3+\sqrt3a^2}{2a+\sqrt3}$, $y=\dfrac{2\sqrt3a-1}{2a+\sqrt3}x+\dfrac{2a^3-\sqrt3a^2+a}{2a+\sqrt3}$

(2) $S(a)=\int_0^a x^2dx=\left[\dfrac{x^3}{3}\right]_0^a=\dfrac13a^3$

$x^2=\dfrac{2\sqrt3a-1}{2a+\sqrt3}x+\dfrac{2a^3-\sqrt3a^2+a}{2a+\sqrt3}$, $(2a+\sqrt3)x^2+(-2\sqrt3a+1)x-2a^3+\sqrt3a^2-a=0$

$$\begin{array}{r|l}
 & (2a+\sqrt3)x+2a^2-\sqrt3a+1\\ \hline
x-a & (2a+\sqrt3)x^2+(-2\sqrt3a+1)x-2a^3+\sqrt3a^2-a\\
 & (2a+\sqrt3)x^2+(-2a^2-\sqrt3a)x\\ \hline
 & (2a^2-\sqrt3a+1)x-2a^3+\sqrt3a^2-a\\
 & (2a^2-\sqrt3a+1)x-2a^3+\sqrt3a^2-a\\ \hline
 & 0
\end{array}$$

$(x-a)\{(2a+\sqrt3)x+2a^2-\sqrt3a+1\}=0$ より、

Bのx座標は $\dfrac{-2a^2+\sqrt3a-1}{2a+\sqrt3}$

$T(a)=\dfrac16\left(a-\dfrac{-2a^2+\sqrt3a-1}{2a+\sqrt3}\right)^3=\dfrac16\left(\dfrac{2a^2+\cancel{\sqrt3a}+2a^2-\cancel{\sqrt3a}+1}{2a+\sqrt3}\right)^3=\dfrac16\left(\dfrac{4a^2+1}{2a+\sqrt3}\right)^3$

$\dfrac{T(a)}{S(a)}=\dfrac12\left(\dfrac{4a^2+1}{2a^2+\sqrt3a}\right)^3=\dfrac12\left(\dfrac{4+\frac{1}{a^2}}{2+\frac{\sqrt3}{a}}\right)^3$, $\lim_{a\to\infty}\dfrac{T(a)}{S(a)}=4$