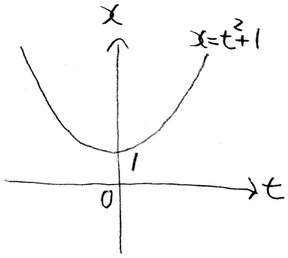
東大理科　1961前期　⑤

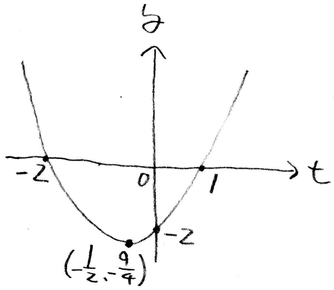


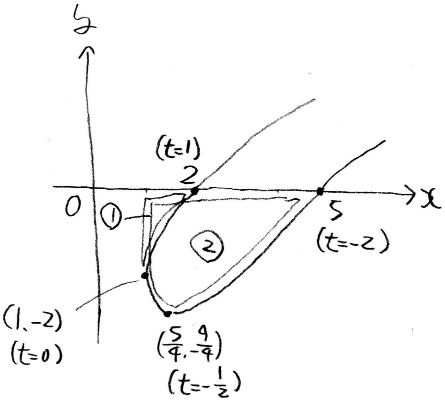
$$y = t^2+t+\frac{1}{4}-\frac{9}{4}$$
$$=\left(t+\frac{1}{2}\right)^2-\frac{9}{4}$$

$y=0$のとき

$(t+2)(t-1)=0$

$t=-2,\ 1$





$t<-2$のとき、xは単調減少
　$y>0$、yは単調減少

$-2<t<-\frac{1}{2}$のとき　xは単調減少
　$y<0$、yは単調減少

$-\frac{1}{2}<t<0$のとき　xは単調減少
　$y<0$、yは単調増加

$0<t<1$のとき　xは単調増加
　$y<0$、yは単調増加

$t>1$のとき　xは単調増加
　$y>0$、yは単調増加

(x, y)の軌跡は左図のようになる。

①の面積は

$$-\int_1^2 y\,dx = -\int_0^1 y(t)\frac{dx(t)}{dt}dt = -\int_0^1 (t^2+t-2)2t\,dt$$

$$=-\int_0^1(2t^3+2t^2-4t)dt = -2\left[\frac{t^4}{4}+\frac{t^3}{3}-\cancel{2}\frac{t^2}{\cancel{2}}\right]_0^1$$

$$=-2\left(\frac{1}{4}+\frac{1}{3}-1\right) = -2\,\frac{3+4-12}{\cancel{12}6} = \frac{5}{6}$$

②の面積は

$$-\int_1^5 y\,dx = -\int_0^{-2} y(t)\frac{dx(t)}{dt}dt = \int_{-2}^0 (t^2+t-2)2t\,dt$$

$$=\int_{-2}^0(2t^3+2t^2-4t)dt = 2\left[\frac{t^4}{4}+\frac{t^3}{3}-\cancel{2}\frac{t^2}{\cancel{2}}\right]_{-2}^0$$

$$=-2\left(\frac{\cancel{16}}{\cancel{4}}-\frac{8}{3}-\cancel{4}\right) = \frac{16}{3}$$

よって求める面積は　$\frac{16}{3}-\frac{5}{6}=\frac{32-5}{6}=\frac{9}{2}$