東大理科　1987前期　⑤

$n=2$のときを考える.

$(x_1-y_2)^2+(x_2-y_1)^2=x_1^2-2x_1y_2+y_2^2+x_2^2-2x_2y_1+y_1^2=(x_1-y_1)^2+(x_2-y_2)^2+2x_1y_1+2x_2y_2-2x_1y_2-2x_2y_1$

$=(x_1-y_1)^2+(x_2-y_2)^2+2x_1(y_1-y_2)-2x_2(y_1-y_2)=(x_1-y_1)^2+(x_2-y_2)^2+2(x_1-x_2)(y_1-y_2)\geqq(x_1-y_1)^2+(x_2-y_2)^2$ より

題意は成り立つ　—①

$n=k$のとき題意は成り立つと仮定する.

$n=k+1$のときを考える.

(i) $z_{k+1}=y_{k+1}$ のとき.

$$\sum_{j=1}^{k+1}(x_j-z_j)^2=\underbrace{\sum_{j=1}^{k}(x_j-z_j)^2}_{\text{この中に}y_1,y_2\cdots y_k\text{がある}}+(x_{k+1}-y_{k+1})^2\geqq\sum_{j=1}^{k}(x_j-y_j)^2+(x_{k+1}-y_{k+1})^2=\sum_{j=1}^{k+1}(x_j-y_j)^2$$

(ii) $z_l=y_{k+1}$ $(l\neq k+1)$ のとき.

$$\sum_{j=1}^{k+1}(x_j-z_j)^2=(x_1-z_1)^2+(x_2-z_2)^2+\cdots+(x_l-y_{k+1})^2+\cdots+(x_k-z_k)^2+(x_{k+1}-z_{k+1})^2$$

$$=(x_1-z_1)^2+(x_2-z_2)^2+\cdots+x_l^2-2x_ly_{k+1}+y_{k+1}^2+\cdots+(x_k-z_k)^2+x_{k+1}^2-2x_{k+1}z_{k+1}+z_{k+1}^2$$

$$=\underbrace{(x_1-z_1)^2+(x_2-z_2)^2+\cdots+(x_l-z_{k+1})^2+\cdots+(x_k-z_k)^2}_{\text{この中に}y_1,y_2\cdots y_k\text{がある}}+(x_{k+1}-y_{k+1})^2+2x_lz_{k+1}+2x_{k+1}y_{k+1}-2x_ly_{k+1}-2x_{k+1}z_{k+1}$$

$$\geqq\sum_{j=1}^{k}(x_j-y_j)^2+(x_{k+1}-y_{k+1})^2+2x_l(z_{k+1}-y_{k+1})-2x_{k+1}(z_{k+1}-y_{k+1})$$

$$=\sum_{j=1}^{k+1}(x_j-y_j)^2+2(x_l-x_{k+1})(z_{k+1}-y_{k+1})\geqq\sum_{j=1}^{k+1}(x_j-y_j)^2$$

よって題意は成り立つ　—②

①②より.数学的帰納法により.題意は成り立つ