東大理科　1993前期　⑥

(1) $\frac{dx}{dt} = -2\sin t - \sin 2t \cdot 2$　$\frac{dy}{dt} = \cos 2t \cdot 2$

速度ベクトルの大きさの2乗を$V(t)$とおく.

$V(t) = 4\sin^2 t + 8\sin t \sin 2t + 4\sin^2 2t + 4\cos^2 2t$

$\frac{V(t)}{4} = 1 - \cos^2 t + 4(1-\cos^2 t)\cos t + 1 = -4\cos^3 t - \cos^2 t + 4\cos t + 2$

$f(x) = -4x^3 - x^2 + 4x + 2$　$(-1 \leqq x \leqq 1)$を考える.

$f'(x) = -12x^2 - 2x + 4$　$f'(x) = 0$のとき　$6x^2 + x - 2 = 0$.　$x = \frac{-1 \pm \sqrt{1+48}}{12} = \frac{-1 \pm 7}{12} = -\frac{2}{3}, \frac{1}{2}$

x	$\cdots$	-1	$\cdots$	$-\frac{2}{3}$	$\cdots$	$\frac{1}{2}$	$\cdots$	1	$\cdots$
$f'(x)$	$-$	$-$	$-$	0	$+$	0	$-$	$-$	$-$
$f(x)$	$\searrow$	1	$\searrow$	$\frac{2}{27}$	$\nearrow$	$\frac{13}{4}$	$\searrow$	1	$\searrow$

$f(x)$の増減表は左図のようになる.

$f(x)$の最大値は$\frac{13}{4}$, 最小値は$\frac{2}{27}$

$V(t)$の最大値は13, 最小値は$\frac{8}{27}$

速度ベクトルの大きさの最大値は$\sqrt{13}$, 最小値は$\frac{2\sqrt{2}}{3\sqrt{3}} = \frac{2\sqrt{6}}{9}$

$f(-1) = 4 - 1 - 4 + 2 = 1$

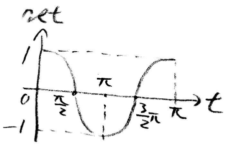
$f(-\frac{2}{3}) = \frac{32}{27} - \frac{4}{9} - \frac{8}{3} + 2 = \frac{32 - 12 - 72 + 54}{27} = \frac{2}{27}$

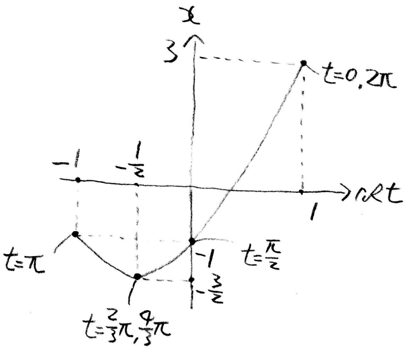
$f(\frac{1}{2}) = -\frac{1}{2} - \frac{1}{4} + 2 + 2 = \frac{-2 - 1 + 16}{4} = \frac{13}{4}$

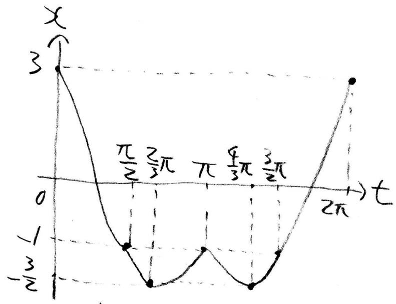
$f(1) = -4 - 1 + 4 + 2 = 1$

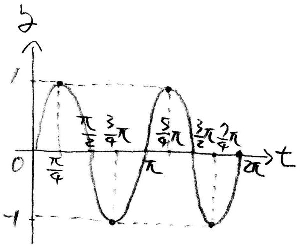
(2) $x = 2\cos t + \cos^2 t - (1 - \cos^2 t) = 2(\cos^2 t + \cos t + \frac{1}{4}) - \frac{1}{2} - 1 = 2(\cos t + \frac{1}{2})^2 - \frac{3}{2}$

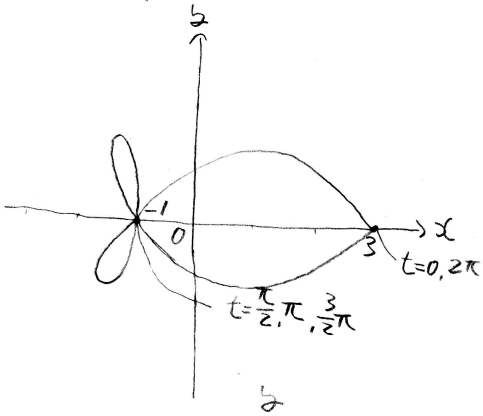
$x = 2\cos^2 t + 2\cos t - 1$











Pの軌跡は左図のようになり.

Pは$t = \frac{\pi}{2}, \pi, \frac{3}{2}\pi$のとき

$(-1, 0)$を通過する.

$t = \frac{\pi}{2}$のとき$\left(\frac{dx}{dt}, \frac{dy}{dt}\right) = (-2, -2)$

$t = \pi$のとき$\left(\frac{dx}{dt}, \frac{dy}{dt}\right) = (0, 2)$

$t = \frac{3}{2}\pi$のとき$\left(\frac{dx}{dt}, \frac{dy}{dt}\right) = (2, -2)$

であるから

速度ベクトルは左図のようになる.