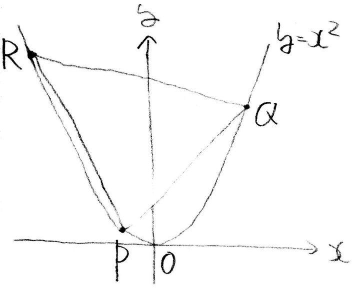
東大理科　2004前期　①




$y=x^2$と$y=\sqrt{2}x+k$の交点を$P(p, p^2)$, $Q(q, q^2)$ $(p<q)$とする.

$x^2-\sqrt{2}x-k=0$と解と係数の関係により $p+q=\sqrt{2}$, $pq=-k$

$2+4k>0$. $k>-\frac{1}{2}$ —①

$\overrightarrow{PQ}=(q-p,\ q^2-p^2)=(\sqrt{2}\sqrt{2k+1},\ 2\sqrt{2k+1})=\sqrt{2}\sqrt{2k+1}\,(1, \sqrt{2})$

※ $(q-p)^2=(p+q)^2-4pq=2+4k$. $q-p=\sqrt{2}\sqrt{2k+1}$

$a=\sqrt{2}\sqrt{2k+1}\sqrt{1+2}=\sqrt{6}\sqrt{2k+1}$ —②

Pの座標は $x^2-\sqrt{2}x-k=0$. $x=\frac{\sqrt{2}\pm\sqrt{2+4k}}{2}=\frac{1}{\sqrt{2}}\pm\frac{\sqrt{2k+1}}{\sqrt{2}}$, $x=\frac{1}{\sqrt{2}}-\frac{\sqrt{2k+1}}{\sqrt{2}}$

$\left(\frac{1}{\sqrt{2}}-\frac{\sqrt{2k+1}}{\sqrt{2}}\right)^2=\frac{1}{2}-\sqrt{2k+1}+\frac{2k+1}{2}=k-\sqrt{2k+1}+1$ より $\left(\frac{1}{\sqrt{2}}-\frac{\sqrt{2k+1}}{\sqrt{2}},\ k-\sqrt{2k+1}+1\right)$

$$\overrightarrow{OR}=\overrightarrow{OP}+\begin{pmatrix}\cos 60^\circ & -\sin 60^\circ\\ \sin 60^\circ & \cos 60^\circ\end{pmatrix}\overrightarrow{PQ}=\begin{pmatrix}\frac{1}{\sqrt{2}}-\frac{\sqrt{2k+1}}{\sqrt{2}}\\ k-\sqrt{2k+1}+1\end{pmatrix}+\sqrt{2}\sqrt{2k+1}\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}1\\ \sqrt{2}\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{2}}-\frac{\sqrt{2k+1}}{\sqrt{2}}\\ k-\sqrt{2k+1}+1\end{pmatrix}+\sqrt{2}\sqrt{2k+1}\begin{pmatrix}\frac{1}{2}-\frac{\sqrt{3}}{\sqrt{2}}\\ \frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{\sqrt{2}}-\cancel{\frac{\sqrt{2k+1}}{\sqrt{2}}}+\cancel{\frac{\sqrt{2k+1}}{\sqrt{2}}}-\sqrt{3}\sqrt{2k+1}\\ k-\cancel{\sqrt{2k+1}}+1+\frac{\sqrt{3}\sqrt{2k+1}}{\sqrt{2}}+\cancel{\sqrt{2k+1}}\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{2}}-\sqrt{3}\sqrt{2k+1}\\ k+1+\frac{\sqrt{3}\sqrt{2k+1}}{\sqrt{2}}\end{pmatrix}$$

Rが$y=x^2$上にあるから $k+1+\frac{\sqrt{3}\sqrt{2k+1}}{\sqrt{2}}=6k-\sqrt{6}\sqrt{2k+1}+\frac{7}{2}$, $5k+\frac{5}{2}=\frac{3}{2}\sqrt{6}\sqrt{2k+1}$

※ $\left(\frac{1}{\sqrt{2}}-\sqrt{3}\sqrt{2k+1}\right)^2=\frac{1}{2}-\sqrt{6}\sqrt{2k+1}+3(2k+1)=6k-\sqrt{6}\sqrt{2k+1}+\frac{7}{2}$

①より

両辺が正であるから $25k^2+25k+\frac{25}{4}=\frac{9}{4}6(2k+1)$. $100k^2+100k+25=108k+54$. $100k^2-8k-29=0$

$k=\frac{4\pm\sqrt{16+2900}}{100}=\frac{4\pm\sqrt{2916}}{100}=\frac{4\pm 54}{100}$.　　①より $k=\frac{29}{50}$

$4\overline{)2916}=729$　$9\overline{)729}=81$

②より $a=\sqrt{6}\sqrt{\frac{58+50}{50}}=\cancel{\sqrt{6}}^{\sqrt{3}}\frac{6\sqrt{3}}{5\cancel{\sqrt{2}}}=\frac{18}{5}$

$2916=4\cdot9\cdot9\cdot9=(54)^2$

$9\overline{)108}=12$　$108=9\cdot4\cdot3=36\cdot3$