

東大文科 2011前期 (1)

$$f(1)=1 \neq 1 \quad a+b+c+d=1$$

$$f(-1)=-1 \neq 1 \quad -a+b-c+d=-1$$

$$\text{よ、} 2b+2d=0, d=-b, 2a+2c=2, c=-a+1$$

$$\int_{-1}^1 (bx^2+cx+d)dx=1 \neq 1 \quad \left[b\frac{x^3}{3}+c\frac{x^2}{2}+dx \right]_{-1}^1 = \frac{1}{3}b+\frac{1}{2}c+d - \left(-\frac{1}{3}b+\frac{1}{2}c-d \right) = \frac{2}{3}b+2d = -\frac{4}{3}b=1, b=-\frac{3}{4}, d=\frac{3}{4}$$

$$\text{よ、} f(x)=ax^3-\frac{3}{4}x^2+(-a+1)x+\frac{3}{4} \text{ と書ける}$$

$$f'(x)=3ax^2-\frac{3}{2}x-a+1, f''(x)=6ax-\frac{3}{2}, \{f'(x)\}^2=36a^2x^2-18ax+\frac{9}{4}$$

$$I=\int_{-1}^1 \left[36a^2\frac{x^3}{3}-18a\frac{x^2}{2}+\frac{9}{4}x \right] \frac{1}{2} = 12a^2\frac{1}{8}-9a\frac{1}{4}+\frac{9}{4}\frac{1}{2} - \left(-12a^2-9a-\frac{9}{4} \right) = \frac{3}{2}a^2-\frac{9}{4}a+\frac{9}{8}+12a^2+9a+\frac{9}{4}$$

$$= \frac{27}{2}a^2+\frac{27}{4}a+\frac{27}{8} = \frac{27}{2}\left(a^2+\frac{1}{2}a+\frac{1}{16}\right) - \frac{27}{2}\cdot\frac{1}{16}+\frac{27}{8} = \frac{27}{2}\left(a+\frac{1}{4}\right)^2+\frac{81}{32}$$

$$\text{よ、} f(x)=-\frac{1}{4}x^3-\frac{3}{4}x^2+\frac{5}{4}x+\frac{3}{4} \text{ のとき } I \text{ は最小値 } \frac{81}{32} \text{ をとる}$$